
THE TIPPING POT PROBLEM

P. T. Arveson
Solar Household Energy, Inc.

Abstract

Burns caused by scalding due to the tipping of a pot of hot water are a common cause of injuries in developing countries, where food is often cooked over small cookstoves that can be top heavy and inadequately supported. An international standard for field testing of cookstoves is under development (ISO-19869). In the course of drafting the safety assessment of this standard, questions arose about how to quantify the risks of kitchen accidents. A quantitative definition of cooking pot stability requires knowing the angle at which a pot becomes unstable. This article describes a calculation of stability for the simple case of a cylindrical container partially filled with liquid.

Introduction

ACCORDING TO THE WORLD HEALTH ORGANIZATION burns are a global public health problem, accounting for many injuries and an estimated 180,000 deaths annually. The majority of these occur in low-income countries, and the victims are primarily women and children. For instance, in India over 1 million people are moderately or severely burned each year. But burns also are a common type of injury in developed countries. Accidents such as burns and scalds are most commonly related to cooking [1, 2].

Over the past five years an international team of consultants in various fields have been developing ISO standards for the testing and evaluation of cookstoves [3, 4]. “Cookstove” is the term the team used to describe simple, low-cost devices or structures for cooking. They are typically fueled by wood, kerosene (paraffin), or bottled gas. These devices are prevalent in less-developed countries. Many governmental and nongovernmental organizations are involved in the research, design, and distribution of improved cookstoves, and this effort has been promoted and coordinated by the Global Alliance for Clean Cookstoves [5, 6].

Recently the ISO TC-285 Working Group 3 completed a Draft International Standard for field testing; this document will be released for public review in a few months. Among other tests it includes protocols for the evaluation of the safety of cookstoves in household settings. In addition

to kitchen observations by trained inspectors, the protocol calls for a “stability test” of cookstoves with a large pot of water. Observations of many cookstove settings indicates that quite often this arrangement is top heavy and prone to tipping or spilling. Figure 1 shows an example of this situation.



Figure 1: Typical cookstove with pot suspended at three points.
(Photograph courtesy of C. Pemberton-Pigott).

The ISO standards should encourage designers to consider how to improve the stability of cooking vessels and the cookstove. In an effort to quantify the stability requirements of a design, it would be helpful to define the stability criterion for the general case.

The problem of the stability of a tipped cylinder or box has been worked out previously (for example, [7]), but here we will include the mass of a cylindrical container and review the calculations of stability. Clearly this calculation could be applied equally to cooking pots, paint cans, 55-gallon drums, or any other cylindrical containers with thin walls compared to their diameter.

The object of interest is a cylindrical container partially filled with a liquid. The container is assumed to have thin walls relative to its diameter and is made of a material with density ρ . The container wall thickness is t . The liquid has a density of δ . The cylinder has radius a and the height of the container is h ; the height of the liquid in the container when level is j .

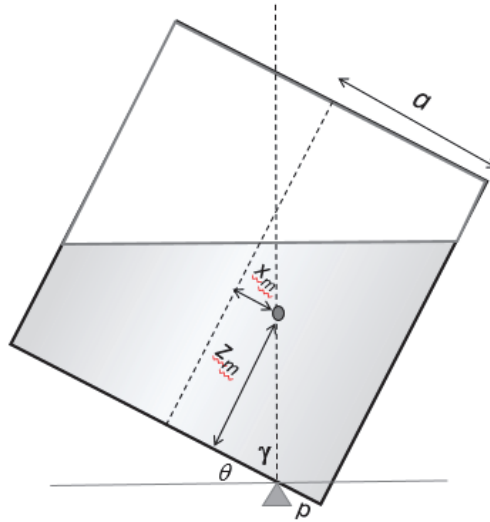


Figure 2: Condition for critical stability of a container of liquid.

From Figure 2 it is evident that the container will tip over when the center of mass extends beyond a vertical line from the pivot point. Note that the pivot point could be placed at less than the radius a of the container, *i.e.* $(a - p)$. This is actually a common – and dangerous -- situation with cookstoves, where the pot support structure or contact points has a smaller effective radius than cooking vessel used.

Tipping over will occur when angles $\theta + \gamma$ exceed $\pi/2$. So the criterion for stability is given by Equation (1):

$$\theta < \frac{\pi}{2} - \arctan\left(\frac{z_m}{a - p - x_m}\right) \quad (1)$$

where θ is the tipping angle and x_m and z_m are coordinates of the total center of mass. (This calculation is restricted to angles such that the height of the container $h > j + 2a \tan \theta$ so that the liquid does not spill out and the liquid level does not intersect the bottom of the container).

The calculation of the position of the center of mass can be facilitated by orienting the geometry so as to simplify the problem: by inspection, symmetry can best be exploited by setting the origin at the center of the base of the cylindrical container. The problem is simplified further by dividing the calculations into four parts: the cylindrical wedge of liquid, the cylinder-

shaped portion of liquid, the cylinder walls and the circular base. The centers of mass of each of these parts will be calculated and then combined to yield the total center of mass in rectangular coordinates as x_m , y_m , z_m . Figure 3 illustrates this geometry.

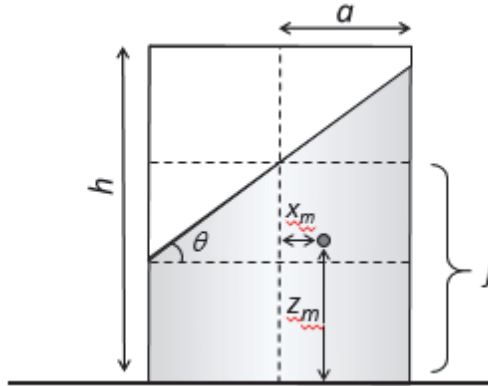


Figure 3: Geometry for calculation of center of mass.

Part 1 – liquid cylindrical wedge

The mass of liquid in the wedge can be calculated by means of the triple integral shown in Equation (2):

$$m_1 = \delta \int_{-a}^a \int_{-qx}^{qx} \int_0^{(a+x)\tan\theta} dz dy dx = \delta \pi a^3 (\tan\theta)^2 \quad (2)$$

where $qx = \sqrt{a^2 - x^2}$. Although the volume (and hence mass) of this part could be found by inspection (it is half of the volume of a cylinder of equal height), the construction of the triple integral in rectangular coordinates will be useful for center of mass calculations.

The center of mass of each part is given by the first moment divided by the mass of the part. The calculation must be done for each of the coordinate axes. The cylinder wedge moment relative to the xy plane (the z -axis) is given by Equation (3):

$$M_{xy1} = \delta \int_{-a}^a \int_{-q}^q \int_0^{(a+x)\tan\theta} z dz dy dx = \frac{5}{8} \delta \pi a^4 (\tan\theta)^2. \quad (3)$$

Although not needed for the final calculations, it is of interest to calculate the coordinates of the centers of mass of the wedge. Dividing M_{xy1} by the mass m_1 yields the z -coordinate given by Equation (4):

$$z_1 = \frac{5}{8} a \tan \theta . \quad (4)$$

The moment in the yz -plane (parallel to the x -axis) is given by Equation (5):

$$M_{yz1} = \delta \int_{-a}^a \int_{-q}^q \int_0^{(a+x)\tan\theta} x dz dy dx = \frac{\pi}{4} \delta a^4 \tan \theta . \quad (5)$$

Dividing this result by m_1 yields the x -coordinate of the center of mass of this part given by Equation (6):

$$x_1 = \frac{a}{4} . \quad (6)$$

(This result seems counterintuitive; it says that the center of mass in the x direction does not depend on tip angle θ . This implies that as θ increases, the mass in the wedge increases in the same proportion, so the θ -dependences cancel each other out).

The moment in the xz -plane (parallel to the y -axis) is zero in all of these calculations due to the symmetry in this plane.

The calculation of the center of mass for the entire filled container will result in different coordinates from these, based on the sum of moments of each part.

Part 2 – the liquid cylinder

The mass in the liquid cylinder is derived from the geometry of a right circular cylinder, but its height is reduced by an amount depending on the tip angle θ , so the resulting mass is given by Equation (7):

$$m_2 = \delta \pi a^2 (j - a \tan \theta) \quad (7)$$

The moment M_{xy2} in the z -axis direction is calculated by the triple integral shown in Equation (8):

$$M_{xy2} = \delta \int_0^{j-a \tan \theta} \int_{-a}^a \int_{-qy}^{qy} z dx dy dz = \frac{\pi}{2} \delta a^2 (j - a \tan \theta)^2 \quad (8)$$

where $qy = \sqrt{a^2 - y^2}$. The moments M_{yz2} and M_{xz2} are zero by symmetry.

Dividing M_{xy2} by m_2 yields the z -coordinate of the center of mass of this part as Equation (9):

$$z_2 = \frac{1}{2} (j - a \tan \theta). \quad (9)$$

Part 3 - Container walls

The container's cylindrical side is assumed to be thin compared to the radius a , so the wall volume is equivalent to the area times thickness t . Then the mass of the wall is given by Equation (10):

$$m_3 = 2\rho\pi aht. \quad (10)$$

By inspection it is seen that the center of mass of the cylindrical wall is at $h/2$, so the moment in the xy -plane is given by Equation (11):

$$M_{xy3} = \frac{h}{2} m_3. \quad (11)$$

The moments in the other two planes are zero by symmetry.

Part 4 – Container base

The base is a circular disk with mass given by Equation (12):

$$m_4 = \rho\pi a^2 t. \quad (12)$$

Since the base is centered on the origin of coordinates, the moments of the base are zero in all three planes.

Total moments and center of mass

The center of mass of a collection of objects is given by the sum of moments divided by the total mass. This calculation must be done separately for each of the three coordinates. The total mass of the four parts is given by Equation (13):

$$m = m_1 + m_2 + m_3 + m_4 = \delta\pi a^3 \tan\theta + \delta\pi a^2 (j - a \tan\theta) + 2\rho\pi a h t + \rho\pi a^2 t. \quad (13)$$

For the x -coordinate of the center of mass, we have Equation (14):

$$x_m = \frac{M_{yz}}{m} = \frac{\delta\pi a^4 \tan\theta}{4[\delta\pi a^3 \tan\theta + a^2 \delta\pi (j - a \tan\theta) + a^2 \pi \rho t + 2ah\pi \rho t]} \quad (14)$$

since the three other moments in the yz -plane are zero due to symmetry.

The z -coordinate of the center of mass is more complicated, since three of the parts have non-zero moments. It is given by Equation (15):

$$z_m = \frac{M_{xy}}{m} = \frac{\frac{5}{8}\delta\pi a^4 (\tan\theta)^2 + \frac{1}{2}\delta a^2 (j - a \tan\theta)^2 + \rho\pi a h^2 t}{\delta\pi a^3 \tan\theta + a^2 \delta\pi (j - a \tan\theta) + a^2 \pi \rho t + 2ah\pi \rho t}. \quad (15)$$

Finally, due to symmetry, $y_m = 0$.

Critical tipping angle

It is now possible to calculate the critical tipping angle θ_c using the calculated coordinates of the center of mass shown by Equation (16):

$$\theta_c = \frac{\pi}{2} - \arctan \frac{z_m}{a - p - x_m}. \quad (16)$$

Note that because the liquid in the container moves as θ changes, the values of x_m and z_m are functions of θ . Hence Equation (16) does not give a unique solution for the critical angle. Therefore an iterative procedure must be used to solve for the critical tipping angle. An initial guess for θ_c is inserted into the equations for x_m and z_m , and Equation (16) is calculated. Based on this answer, a closer estimate is made, and the calculation is repeated until the desired degree of tolerance is achieved. (Of course, this process could be automated).

Table 1 shows an example of iterated results. The test vessel was a 3-liter polyethylene pitcher with $a = 7.75$ cm, $j = 18$, $h = 25$, $\delta = 1$, $\rho = 1$.

θ , input	θ , calculated
30	40.1509
35	38.9447
36	38.7424
37	38.5571
38	38.3906
38.4	38.3297
38.41	38.3282
38.37	38.3342
38.35	38.3371
38.34	38.3386

Table 1. Example of iterated results showing convergence to .01 degree within 10 iterations.

Stability of a cooking system may be defined as its closeness to the critical angle θ_c when a large full container of water is placed on the cooking surface.

Generalizations

The basic method of calculating stability for a container could be generalized in several ways, for example:

- Many vessels are not right circular cylinders but are rounded or tapered in shape. The triple integral method could be extended to any shape that is a body of revolution.
- The method could be extended to the case where the liquid level cuts the flat bottom of the container. The calculation for the centroid is much more complicated in this case, but it has been done [8] and this would provide a solution for large tipping angles.

Conclusion

Evidence from the field indicates that one of the most common burn hazards in the world is the instability of cooking pots and/or cookstoves. This article has reviewed the calculation for stability of a cylindrical container as a way of quantifying what “instability” means, or when the combination of a vessel and a cookstove may be “top heavy”. Stability of a particular cooking system may be assessed by measuring the critical angle when a large container of (cold) water is placed on the cookstove. Designers of cookstoves can reduce the risk of this hazard by including a wide base, a wide pot holder, and/or a skirt around the cooking pot, and by increasing the number of pot supports, increasing the effective contact radius.

Acknowledgements

This problem was first posed to the author by Crispin Pemberton-Pigott, a Canadian manager of global field programs for cookstoves. Thanks also are due to Stephen Wolfram, who distributed a free version of *Mathematica* for the Raspberry Pi computer, which was used to check some of the calculations here.

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Bio

Paul Arveson is Director of Research for Solar Household Energy, Inc., a nonprofit organization based in Washington, DC. His address: 6902 Breezewood Terrace, Rockville, MD 20852. Email paularveson@gmail.com . He is a retired physicist, and is currently serving on the board of the Washington Academy of Sciences.