THE TIPPING POT PROBLEM

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Abstract

Burns caused by scalding due to the tipping of a pot of hot water are a common cause of injuries in developing countries, where food is often cooked over small cookstoves that can be top heavy and inadequately supported. An international standard for field testing of cookstoves is under development (ISO-19869). In the course of drafting the safety assessment of this standard, questions arose about how to quantify the risks of kitchen accidents. A quantitative definition of cooking pot stability requires knowing the angle at which a pot becomes unstable. This article describes a calculation of stability for the simple case of a cylindrical container partially filled with liquid.

Introduction

ACCORDING TO THE WORLD HEALTH ORGANIZATION, burns are a global public health problem, accounting for many injuries and an estimated 180,000 deaths annually. The majority of these occur in low-income countries, and the victims are primarily women and children. For instance, in India, over 1 million people are moderately or severely burned each year. But burns also are a common type of injury in developed countries. Accidents such as burns and scalds are most commonly related to cooking [1,2].

Over the past five years, an international team of consultants in various fields have been developing ISO standards for the testing and evaluation of cookstoves [3,4]. "Cookstove" is the term the team used to describe simple, low-cost devices or structures for cooking. They are typically fueled by wood, kerosene (paraffin), or bottled gas. These devices are prevalent in less-developed countries. Many governmental and nongovernmental organizations are involved in the research, design and distribution of improved cookstoves, and this effort has been promoted and coordinated by the Global Alliance for Clean Cookstoves [5,6].

Recently the team has completed a Draft International Standard for field testing; this document will be released for public review in a few months. Among other tests, it includes protocols for the evaluation of safety of cookstoves in household settings. In addition to kitchen observations by trained inspectors, the protocol calls for a "stability test" of cookstoves with a large pot of water. Observations of many cookstove settings indicates that quite often this arrangement is top heavy, and prone to tipping or spilling.

The ISO standards should encourage designers to consider how to improve the stability of cooking vessels and the cookstove. In an effort to quantify the stability requirements of a design, it would be helpful to define the stability criterion for the general case.

The problem of the stability of a tipped cylinder or box has been worked out previously (for example, [7]), but here we will include the mass of a cylindrical container and review the calculations of stability. Clearly this calculation could be applied equally to cooking pots, paint cans, 55-gallon drums, or any other cylindrical containers with thin walls compared to their diameter.

The object of interest is a cylindrical container partially filled with a liquid. The container is assumed to have thin walls relative to its diameter and is made of a material with density ρ. The container wall thickness is t. The liquid has a density of δ. The cylinder has radius a and the height of the container is h; the height of the liquid in the container when level is j.

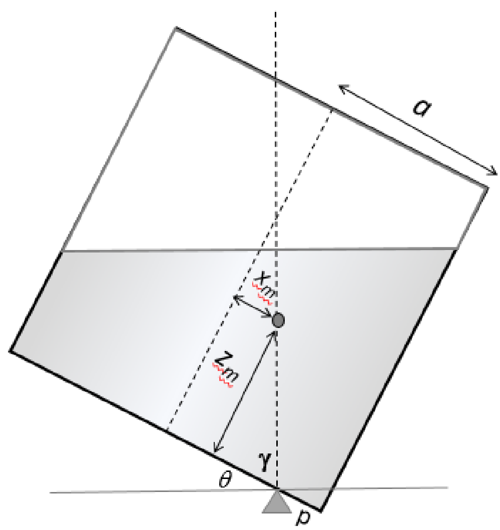

Figure 1.

From Figure 1, it is evident that the container will tip over when the center of mass extends beyond a vertical line from the pivot point. Note that the pivot point could be placed at less than the radius a of the container, *i.e.* $(a - p)$. This is actually a common – and dangerous -- situation with cookstoves, where the stove has a smaller diameter than cooking vessels that are commonly used.

Tipping over will occur when angles $\theta + \gamma$ exceed $\pi/2$. So the criterion for stability is

$$\theta < \frac{\pi}{2} - arctan\left(\frac{z_m}{a-p-x_m}\right) \qquad (1)$$

where θ is the tipping angle and x_m and z_m are coordinates of the total center of mass. (This calculation is restricted to angles such that the height of the container $h > j + a \tan \theta$ so that the liquid does not spill out and the water level does not cut the bottom of the container).

The calculation of center of mass can be facilitated by orienting the geometry so as to simplify the problem; by inspection symmetry can best be exploited by setting the origin at the center of the base of the cylindrical container. The problem is simplified further by dividing the calculations into four parts: the cylindrical wedge of liquid, the cylinder-shaped portion of liquid, the cylinder walls and the circular base. The centers of mass of each of these parts will be calculated and then combined to yield the total center of mass in rectangular coordinates as x_m, y_m, z_m. Figure 2 is an illustration of this setup.

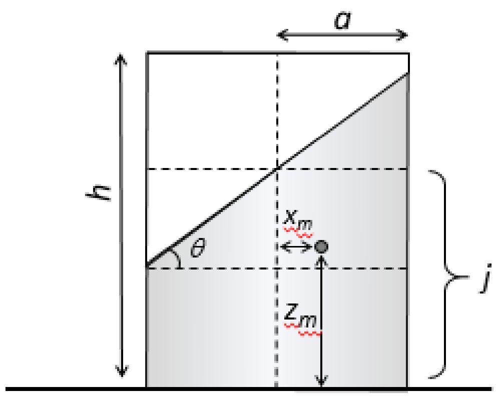

Figure 2.

Part 1 – liquid cylindrical wedge

The mass of liquid in the wedge can be calculated by means of a triple integral:

$$m_1 = \delta \int_{-a}^{a} \int_{-qx}^{qx} \int_{0}^{(a+x)tan\theta} dzdydx \ = \delta\, \pi\, a^3 \tan\theta \qquad (2)$$

where $qx = \sqrt{a^2 - x^2}$. Although the volume (and hence mass) of this part could be found by inspection (it is half of the volume of a cylinder of equal height), the construction of the triple integral in rectangular coordinates will be useful for center of mass calculations.

The center of mass of each part is given by the first moment divided by the mass of the part. The calculation must be done for each of the coordinate axes. The cylinder wedge moment relative to the *xy* plane (the *z*-axis) is given by

$$M_{xy1} = \delta \int_{-a}^{a} \int_{-q}^{q} \int_{0}^{(a+x)tan\theta} z\, dzdydx \quad = \frac{5}{8} \delta\, \pi\, a^4\, (\tan\theta)^2 \qquad (3)$$

Although not needed for the final calculations, it is of interest to calculate the coordinates of the centers of mass of the wedge. Dividing M_{xy1} by the mass m_1 yields the *z*-coordinate:

$$z_1 \;=\; \frac{5}{8}\, a \tan\theta \qquad (4)$$

The moment in the *yz*-plane (parallel to the *x*-axis) is given by:

$$M_{yz1} = \delta \; \int_{-a}^{a}\int_{-q}^{q}\int_{0}^{(a+x)tan\theta} x \; dzdydx \quad = \frac{\pi}{4}\,\delta\; a^4 \tan\theta$$
(5)

Dividing this result by m_1 yields the *x*-coordinate of the center of mass of this part:

$$x_1 \;=\; \frac{a}{4} \qquad (6)$$

(This result seems counterintuitive; it says that the center of mass in the *x* direction does not depend on tip angle θ. This implies that as θ increases, the mass in the wedge increases in the same proportion, so the θ-dependences cancel each other out).

The moment in the *xz*-plane (parallel to the *y*-axis) is zero in all of these calculations due to the symmetry in this plane.

The calculation of the center of mass for the entire filled container will result in different coordinates from these, based on the sum of moments of each part.

Part 2 – the liquid cylinder

The mass in the liquid cylinder is derived from the geometry of a right circular cylinder, but its height is reduced by an amount depending on the tip angle θ, so the resulting mass is:

$$m_2 = \;\delta\,\pi\, a^2\;(j - a\tan\theta) \qquad (7)$$

The moment M_{xy2} in the z-axis direction is calculated by the triple integral

$$M_{xy2} = \delta\; \int_{0}^{j-a\;tan\theta}\int_{-a}^{a}\int_{-qy}^{qy} z\; dxdydz \quad = \frac{\pi}{2}\,\delta\; a^2 (j - a\; tan\;\theta)^2$$
(8)

where $qy = \sqrt{a^2 - y^2}$. The moments M_{yz2} and M_{xz2} are zero by symmetry.

Dividing M_{xy2} by m_2 yields the coordinate of the center of mass of this part as

$$z_2 = \frac{1}{2}\,(j - a\,tan\,\theta) \qquad (9)$$

Part 3 - Container walls

The container's cylindrical side is assumed to be thin compared to the radius a, so the wall volume is equivalent to the area times thickness t. Then the mass of the wall is

$$m_3 = 2\,\rho\,\pi\,a\,h\,t \qquad (10)$$

By inspection it is seen that the center of mass of the cylindrical wall is at $h/2$, so the moment in the xy-plane is

$$M_{xy3} = \frac{h}{2}\,m_3\ . \qquad (11)$$

The moments in the other two planes are zero by symmetry.

Part 4 – Container base

The base is a circular disk with mass

$$m_4 = \rho\,\pi\,a^2\,t \qquad (12)$$

Since the base is centered on the origin of coordinates, the moments of the base are zero in all three planes.

Total moments and center of mass

The center of mass of a collection of objects is given by the sum of moments divided by the total mass. This calculation must be done separately for each of the three coordinates. The total mass of the four parts is

$$m = m_1 + m_2 + m_3 + m_4$$

$$= \delta\pi a^3 \tan\theta + \delta\pi a^2\,(j - a\,tan\theta) + 2\,\rho\pi a\,h\,t + \rho\pi a^2 t\ . \qquad (13)$$

So for the x-coordinate of the center of mass, we have

$$x_m = \frac{M_{yz}}{m} = \frac{\delta\pi a^4 \ tan\theta}{4(\delta\pi a^3 \tan\theta + a^2\delta\pi(j - a\ tan\theta) + a^2\pi\rho t + 2\ a\ h\ \pi\rho t)} \qquad (14)$$

since the three other moments in the *yz*-plane are zero due to symmetry.

The *z*-coordinate of the center of mass is more complicated, since three of the parts have non-zero moments:

$$z_m = \frac{M_{xy}}{m} = \frac{\frac{5}{8}\delta\pi a^4\ (\tan\theta)^2 + \frac{1}{2}\delta\pi a^2(j - a\ tan\theta)^2 + \rho\pi a h^2 t}{\delta\pi a^3 \tan\theta + a^2\delta\pi(j - a\ tan\theta) + a^2\pi\rho t + 2\ a\ h\ \pi\rho t} \qquad (15)$$

Finally, due to symmetry, $y_m = 0$.

Critical tipping angle

It is now possible to calculate the critical tipping angle θ_c using the calculated center of mass coordinates:

$$\theta_c = \frac{\pi}{2} - arctan\left(\frac{z_m}{a - p - x_m}\right). \qquad (16)$$

Note that this equation does not give a unique solution for the critical angle, but rather a solution in terms of both x_m and z_m. The tipping angle could exist for a center of mass located anywhere along the vertical line from the pivot point. This means that in practice an iterative procedure must be used to identify the critical tipping angle. An initial guess θ is inserted into the equations for x_m and z_m, and equation (16) is calculated. Based on this answer, a closer estimate is made, and the calculation is repeated until the desired degree of tolerance is achieved. (Of course this process could be automated).

Stability of a cooking system may be defined as its closeness to the critical angle θ_c when a large full container of water is placed on the cooking surface.

Generalizations

The basic method of calculating stability for a container could be generalized in several ways, for example:

- Many vessels are not right circular cylinders but are rounded or tapered in shape. The triple integral method could be extended to any shape that is a body of revolution.
- The method could be extended to the case where the liquid level cuts the flat bottom of the container. The calculation for the centroid is much more complicated in this case, but it has been done [8] and this would provide a solution for large tipping angles.

Conclusion

Evidence from the field indicates that one of the most common burn hazards in the world is the instability of cooking pots and/or cookstoves. This article has reviewed the calculation for stability of a cylindrical container as a way of quantifying what "instability" means, or when the combination of a vessel and a cookstove may be "top heavy". Stability of a particular cooking system may be assessed by measuring the critical angle when a large container of (cold) water is placed on the cookstove. Designers of cookstoves can reduce the risk of this hazard by including a wide base, a wide pot holder, and/or a skirt around the cooking pot.

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